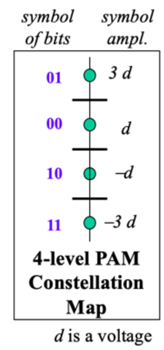
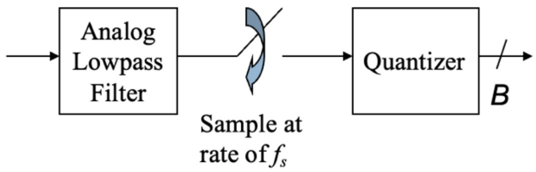
[10:33] Quantization (Lecture 8)



- Amplitude quantization used in several places
  - Decode received symbol amplitude in PAM/QAM to the nearest transmitted symbol amplitude and hence to a symbol of bits
  - Analog-to-digital (A/D) converter for microphone/antenna



- A/D converters have tradeoff between sampling rate f_s and number of bits B
  - Increased f_s reduces sampling time T_s to complete quantization process
  - Conversely, as the number of bits B increases, the sampling rate would decrease.

A/D Converter	Channels	Bits/sample	Sampling rate	Signal Bandwidth
NI PXIe-5186	2	8	12.5 GSPS	5 GHz
NI PXI-5154	2	8	2 GSPS	1 GHz
NI PXIe-5122	1	14	100 MSPS	
NI PXI-5192	1	16	15 MSPS	
NI PXI-5192	1	24	500 KSPS	

  - Sampling rate (conversion rate) in samples per second (SPS) instead of Hz
- One quantizer circuit implementation uses B stages in cascade where each stage is a bank of comparators, and the total delay is the sum of the delays through each stage

[10:40] Uniform amplitude Quantization (Lecture Slide 8-5)

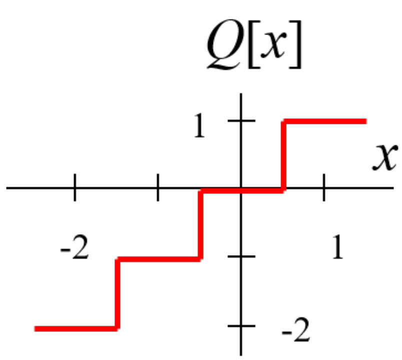
- Midtread (round to nearest integer) vs. midrise (round with offset)

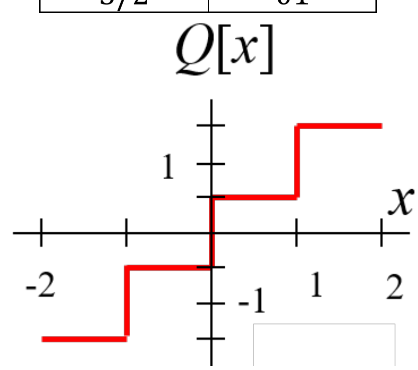
Midtread

Level	Bits	2s comp.
-2	00	10
-1	01	11
0	10	00
1	11	11

Midrise

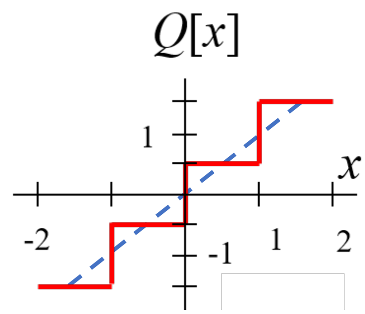
Level	Bits
-3/2	10
-1/2	11
1/2	00
3/2	01





$$\text{Step Size: } \Delta = \frac{\max - \min}{2^B - 1}$$

[10:45] Quantization error analysis



- Assume midrise quantization with B bits:
$$m \to Q_B[\cdot] \to v$$
- Assume input signal x is uniformly distributed in linear region of quantizer, e.g. between -1.5V and 1.5V on right:
- The number of quantization levels is $L = 2^B$
$$\text{For } L \text{ large enough: } \frac{1}{L-1} \approx \frac{1}{L}$$
$$\Delta = \frac{2m_{\max}}{L-1} \approx \frac{2m_{\max}}{L}$$
- Quantization error $q = Q_B[m] - m = v - m$
    - q is uniformly distributed random variable between $-\Delta/2$ and $+\Delta/2$
    - q has zero mean with variance (power) of
$$\sigma_Q^2 = \underbrace{\frac{\Delta^2}{12}}_{\text{variance for uniform RV}} = \frac{1}{3} m_{\max}^2 2^{-2B}$$
- Additive quantization error (noise) model can be equated to SNR
$$10\log_{10} SNR = 10\log_{10}\left(\frac{P_{av}}{\sigma_Q^2}\right) = 10\log_{10}\left(\frac{3P_{av}}{m_{\max}^2} 2^{2B}\right) = \text{constant} + 6.02 \text{ dB/bit} \times B$$
- System design: choose B to match quantization SNR with thermal noise SNR
    - Using more bits would result in some bits containing no real information
    - Using fewer bits would result in loss of accuracy